Midterm
Solutions

ECE 162C

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Name____

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(20 pts)

1. It is desired to put anti-reflection (AR) coatings on a DFB laser chip. It emits at 1550 nm and is composed of an InGaAsP waveguide and active region clad by InP. It is known that the initial power reflection coefficient from the semiconductor-air cleaved facets is 32%. Assuming that the guided mode reflects just like a plane wave,

(a) What index of refraction is needed for a perfect single-layer AR coating?
(b) What thickness should this layer have?

(a)

amplitudes of incoming and reflected wave must be the same in order for wave to fully destructively interfere with itself, therefore the reflectivity at each interface must be the same:

$$\frac{n_{InGaAsP}-n_{AR}}{n_{InGaAsP}+n_{AR}} = \frac{n_{AR}-n_{Air}}{n_{AR}+n_{Air}}$$

$$r = .32 = \left(\frac{n_{InGaAsP}-n_{Air}}{n_{InGaAsP}+n_{Air}}\right)^2$$; n_{air} =1; now solve for $n_{InGaAsP}$

$$.32 = \left(\frac{n_{InGaAsP} - 1}{n_{InGaAsP} + 1}\right)^2$$

$$n_{InGaAsP} = 3.605$$

now solve first equation for n_{AR}

$$\frac{3.605-n_{AR}}{3.605+n_{AR}} = \frac{n_{AR}-1}{n_{AR}+1}$$

$$(3.605 - n_{AR})(n_{AR} + 1) = (n_{AR} - 1)(3.605 + n_{AR})$$

$$n_{AR} = \sqrt{n_{I\,nGaAsP}n_{Air}} = \sqrt{(3.605)(1)} = \mathbf{1.899}$$

(b)

$$L = \frac{\lambda}{4n} = \frac{1550nm}{4(1.899)} = \mathbf{204.09\ nm}$$

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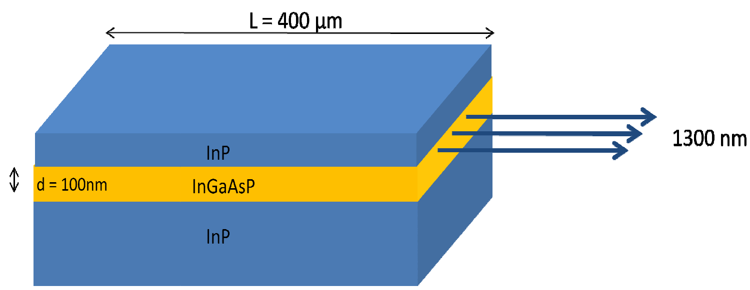
(40 pts)

2. A 400 μm long simple cleaved-facet, double-heterostructure (DH) slab-waveguide laser is made with a 100 nm thick undoped active region InGaAsP layer clad by *n* and *p*-type layers of InP. Laser emission is measured at 1300 nm. The index of the active slab waveguide layer is 3.5 and that of InP is 3.2 at this wavelength. The internal efficiency, η_i, is assumed to be 70 %. The average internal waveguide loss, $<\alpha_i>$, is assumed to be 10 cm^{-1}. The cleaved-facet power reflection coefficient is assumed to be 32%.

(a) What is the effective index of the fundamental transverse mode of the optical waveguide?
(b) What is the confinement factor of this mode to the active region?
(c) What is the threshold material gain, g_{th}?
(d) What is the differential quantum efficiency, η_d?
(e) What is the axial mode spacing, assuming a group index, $n_g = 4.0$.



(a)

$$V = k_0 d\sqrt{n_2^2 - n_3^2} = \frac{2\pi}{1300nm}(100nm)\sqrt{3.5^2 - 3.2^2} = 0.6852$$

$$b = 1 - \frac{\ln\left(\frac{V^2}{2} + 1\right)}{\frac{V^2}{2}} = 1 - \frac{\ln\left(\frac{0.6852^2}{2} + 1\right)}{\frac{0.6852^2}{2}} = 0.10174$$

$$\bar{n}^2 = n_2^2 b + n_3^2(1-b) = (3.5)^2(0.10174) + 3.2^2(1 - 0.10174) = 10.445$$

$$\bar{n}^2 = \sqrt{10.445}$$

$$\boldsymbol{\bar{n} = 3.232}$$

(b)

$$\Gamma \approx \frac{V^2}{2 + V^2} = \frac{(.6852)^2}{2 + (.6852)^2} = \mathbf{0.1901}$$

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(c)

$$\Gamma g_{th} = \alpha_i + \frac{1}{L}\ln\left(\frac{1}{R}\right) = \ 10/cm + \frac{1}{400\times 10^{-4}cm}\ln\left(\frac{1}{.32}\right) = 38.486$$

$$\Gamma g_{th} = 38.486$$

$$(0.1901)g_{th} = 38.486/cm$$

$$\boldsymbol{g_{th} = 202.451/cm}$$

(d)

$$\eta_d = \frac{\eta_i\alpha_m}{\alpha_i + \alpha_m}$$

$$\alpha_m = \frac{1}{L}\ln\left(\frac{1}{R}\right) = \frac{1}{400\times 10^{-4}cm}\ln\left(\frac{1}{.32}\right) = 28.486\ /cm$$

$$\eta_d = \frac{\eta_i\alpha_m}{\alpha_i + \alpha_m} = \frac{0.7(28.486)}{10 + 28.486} = \mathbf{0.518}$$

(e)

$$\Delta\lambda_m = \frac{\lambda^2}{2nL} = \frac{(1300nm)^2}{2(4)(400e3nm)} = \boldsymbol{0.528nm}$$

(20 pts)

3. The absorption coefficient due to the electronic transition, E_{12}, in an unpumped Er-doped fiber amplifier (EDFA) doped with 10^{20} cm^{-3} of Er atoms is found to be 0.1 m^{-1}. It is also found that the linewidth of this absorption is, $\Delta\lambda = 1$ nm.

(a) What is the Einstein "*B* coefficient", if we assume an index of 1.5?

$$g(v_0) \approx (N_2 - N_1)\frac{B_{21}nhv_0}{c\Delta v}$$

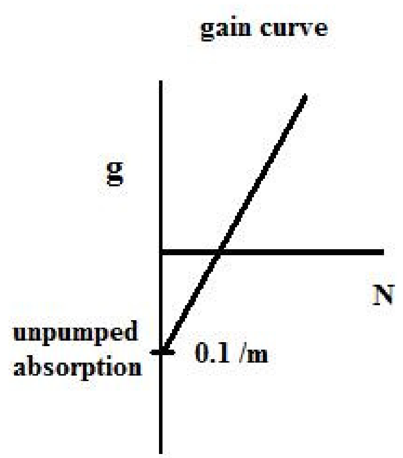
Also,

$$\Delta\lambda = \frac{\lambda}{v}\Delta v$$

Therefore,

$$g(v_0) \approx (N_2 - N_1)\frac{B_{21}nh\lambda_0}{c\Delta\lambda}$$

It is unpumped, therefore $N_2 = 0;\ N_1 = 10^{20}\text{cm}^{-3} = 10^{26}\text{m}^{-3}$



As you can see from the above plot, when a laser is unpumped, it is absorbing photons rather than gaining. Therefore the $g(v_0)$ in this case (unpumped) is the absorption coefficient, and is negative; $g(v_0) = -0.1\ /m$

$$-0.1/m \approx (0 - 10^{26}\text{m}^{-3})\frac{B_{21}(1.5)(6.626\times10^{-34}Js)(1.550\times10^{-6}\ m)}{(3\times10^{8}\ {}^{m}/_{s})(1\times10^{-9}\ m)}$$

$$B_{21} = \frac{(-0.1/m)(3\times10^{8}\ {}^{m}/_{s})(1\times10^{-9}\ m)}{(-10^{26}\text{m}^{-3})(1.5)(6.626\times10^{-34}Js)(1.550\times10^{-6}\ m)}$$

$$B_{21} = \mathbf{1.95\times10^{11}\frac{m^3}{Js^2}}$$

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(20 pts)

4. It is desired to design an GaAs/AlGaAs quantum-well laser, in which there are two well-confined states in the GaAs QW, but also the third state just at cut off, so that its wave function in the SCH region still has a large overlap with the well. We use a well just like the modified homework problem 4.16 with an AlGaAs barrier having 20% Al fraction, an E_g = 1.67 eV, and 2/3 of the band offset in the conduction band.

(a) How wide should the well be for the desired two confined states in the conduction band with a third just almost confined?

$$\Delta E_g = 1.67\ eV - 1.42\ eV = 0.25\ eV = 250\ meV$$

$$\Delta E_C = \frac{2}{3}\Delta E_g = \frac{2}{3}(250meV) = 166.7\ meV$$

From Fig A1.4 in appendix 1 of handouts (Coldren and Corzine)

$n_{max} \approx 2$ for the third state to be just at cut off

$$n_{max}^2 = \frac{\Delta E_C}{E_1^\infty}$$

$$E_1^\infty = \frac{\Delta E_C}{n_{max}^2} = \frac{166.7\ meV}{(2)^2} = 41.675\ meV$$

$$E_1^\infty = 3.76\left(\frac{m_0}{m}\right)\left(\frac{100\ \text{Å}}{L}\right)^2 = 3.76\left(\frac{m_0}{.07m_0}\right)\left(\frac{100\ \text{Å}}{L}\right)^2$$

$$3.76\left(\frac{m_0}{.07m_0}\right)\left(\frac{100\ \text{Å}}{L}\right)^2 = 41.675\ meV$$

$$L^2 = \frac{3.76\left(\frac{m_0}{.07m_0}\right)(100\text{Å})^2}{41.675\ meV} = \frac{3.76\left(\frac{1}{.07}\right)(100\text{Å})^2}{41.675\ meV} = 12888.9\ \text{Å}^2$$

$$L = \sqrt{12888.9\text{Å}} = 113.53\ \text{Å} = 11.353\ nm$$

$$\boldsymbol{L = 11.353\ nm}$$